1. A substance has fcc lattice, molecular weight 60.2 and density 6250 \( kg/m^3 \), calculate its lattice constant.

Solution: Let \( n \) be the number of molecules in a unit cell and \( M \) be the molecular mass, then mass of one molecule = \( M/N \) and total mass of a unit cell = \( \frac{nM}{N} \).

If \( \rho \) be the density of the crystal, then volume of the unit cell \( V = \frac{nM}{\rho N} \).

For fcc (cubic) lattice, \( n = 4 \) and \( V = a^3 \), where \( a \) be the lattice constant. Then

\[
a = (V)^{1/3} = \left(\frac{nM}{\rho N}\right)^{1/3} = \left(\frac{4M}{\rho N}\right)^{1/3}
\]

Here, \( M = 60.2 \), \( \rho = 6250 \( kg/m^3 \), \( N = 6.02 \times 10^{26} \) per kg mole.

\[
\therefore a = \left(\frac{4 \times 60.2}{6250 \times 6.02 \times 10^{26}}\right)^{1/3} = 4 \text{Å}
\]

2. Find the Miller indices of a set of parallel planes which make intercepts in the ratio of 2a:3b on the X and Y axis and are parallel to Z-axis; \( \vec{a}, \vec{b}, \vec{c} \) being primitive vectors to the lattice. Also calculate interplanar distance \( d \) of the plane taking the lattice to be a cube with \( a = b = c = 3 \text{Å} \).

Solution: According to question the intercepts on the X-axis, Y-axis and Z-axis are 2a, 3b and \( \infty \).

The reciprocal of the numbers are \( \frac{1}{2}, \frac{1}{3}, \frac{1}{\infty} \). Multiply by 6, we get: 3, 2 and 0. Thus the miller indices \((h\ k\ l) = (3\ 2\ 0)\).

Interplanar distance \( d = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{3\text{Å}}{\sqrt{3^2+2^2+0^2}} = \frac{3}{\sqrt{13}} \text{Å}\).

3. Find the interplanar distance between \((1\ 1\ 1)\), \((2\ 0\ 0)\) and \((1\ 0\ 1)\) planes of lead. Given that lead has fcc structure with atomic radius 1.75Å.

Hint: For fcc lattice, \( r = \frac{a}{2\sqrt{2}} \). Use \( d(h\ k\ l) = \frac{a}{\sqrt{h^2+k^2+l^2}}\).

Ans. 2.86 Å, 2.47Å, 3.51Å.

4. In a crystal whose primitives are 1.21Å, 1.8Å, 2.0Å. A plane \((2\ 3\ 1)\) cuts an intercept 1.2 Å on the X-axis. Find the corresponding intercepts on the Y and Z-axis.

Hint: Reciprocal of Miller indices : 1/2, 1/3 and 1

Whole numbers : 3,2 and 6

Let a, b and c are the intercepts then:

\[
\frac{3 \times 1.21}{a} = \frac{2 \times 1.8}{b} = \frac{6 \times 2.0}{c}
\]

Here, \( a = 1.2\text{Å} \). Then we get: \( b = 1.2\text{Å} \) and \( c = 4\text{Å} \).

5. The molecular weight of rock salt, a simple cubic crystal is 58.5 kg per kilo-mole and its density is \( 2.16 \times 10^3 \) kg/mole. Taking Avogadro’s number \( 6.02 \times 10^{26} \), calculate the grating space \( d_{100} \) of rock salt. Using this value of lattice constant, calculate the wavelength of X-rays in second order, if angle of diffraction \( \theta = 26^\circ \).

Hint: Mass of rock salt (Nacl) molecule = \( \frac{58.5}{6.02 \times 10^{26}} \)

Volume of the molecule = \( \frac{\text{Mass}}{\text{density}} = \frac{58.5}{6.02 \times 10^{26} \times 2.16 \times 10^3} = 4.5 \times 10^{-29} \text{ m}^3 \)

Volume per atom = \( \frac{4.5 \times 10^{-29}}{2} = 2.25 \times 10^{-29} \text{ m}^3 = d^3 \), where \( d \) is the atomic spacing of the crystal.

\[
\therefore d = 2.82\text{Å}
\]
According to Bragg’s law
\[ 2d \sin \theta = n\lambda, \] where \( n = 2, \theta = 26^\circ \) and \( d = 2.82\text{Å} \). ∴ \( \lambda = 1.24\text{Å} \)

6. The wavelength of prominent X-rays from copper target is 0.1537 nm. The radiation when diffracted from (1 1 1) plane of crystal with FCC structure corresponds to Bragg angle of 19.2°. If the density of crystal is 2698 kg/m³ and atomic weight 26.98 kg/K mole. Calculate Avogadro’s number.

Hint: Let \( a \) be lattice constant, \( n \) be the number of molecules in a unit cell then
\[ a = \left( \frac{nM}{\rho N} \right)^{1/3} = \left( \frac{4 \times 26.98}{2698 N} \right)^{1/3} \]

Using Bragg’s relation \( 2d \sin \theta = \lambda \) for first order, where \( \theta = 19.2^\circ \) and \( \lambda = 0.1537 \text{ nm} = 0.1537 \times 10^{-9} \text{ m} \). This implies \( d = \frac{0.1537 \times 10^{-9}}{2 \times \sin 19.2^\circ} = 2.3 \times 10^{-9} \text{ m} \)

Thus \( d_{111} = \frac{a}{\sqrt{1^2+1^2+1^2}} \Rightarrow a = 0.23 \times 10^{-9} \times \sqrt{3} \)

Therefore
\[ a = \left( \frac{4 \times 26.98}{2698 N} \right)^{1/3} = 0.23 \times 10^{-9} \times \sqrt{3} \]

Or, \( 6.32 \times 10^{-29} = \frac{4 \times 26.98}{2698 N} \)

Or, \( N = 6.3 \times 10^{26} \)

7. A monochromatic beam of x-rays of wavelength 1.24Å is reflected by cubic crystal KCl. Determine the inter planar distances for (100), (110) and (111). Given desnity of KCl= 1980 kg/m³ and molecular weight \( M = 74.5 \text{ kg} \). Avogadro’s number \( N = 6.023 \times 10^{26} \) per K mole.

Hint: Use
\[ a = \left( \frac{nM}{\rho N} \right)^{1/3}, \] where \( n = 4 \) for fcc cubic crystal.

\[ \therefore a = \left( \frac{4 \times 74.5}{1980 \times 6.023 \times 10^{26}} \right)^{1/3} = 6.3 \times 10^{-10} \]

Using \( d_{h k l} = \frac{a}{\sqrt{h^2+k^2+l^2}} \), we get:

\( d_{100} = 6.3 \times 10^{-10} \text{ m}, d_{110} = 4.45 \times 10^{-10} \text{ m}, d_{111} = 3.63 \times 10^{-10} \text{ m} \)

8. From the following data calculate the wavelength of neutron beam and its speed. Spacing between successive (100) planes= 3.84Å, grazing angle 30°, order of Bragg reflection= 1.

Hint: Using the relation \( 2d \sin \theta = n\lambda \), first calculate \( \lambda = 3.84 \text{ Å} \) and then the relation \( \lambda = \frac{h}{mv} \) gives the speed of neutron, where we can use the mass of neutron \( m = 1.67 \times 10^{-27} \text{ kg} \).

\( v = 1.03 \times 10^8 \text{ m/s} \).

9. Show that (i) the reciprocal lattice to the simple cubic (sc) direct space lattice is itself a simple cubic lattice.

(i) The reciprocal lattice to the body centred cubic (iii) The reciprocal lattice to a face centred cubic lattice (fcc) is body centred.

Solution: Suppose the primitive translation vectors of a simple cubic cell be \( \vec{a}, \vec{b} \) and \( \vec{c} \). Then, we can write:

\[ \vec{a} = a\hat{i}, \vec{b} = a\hat{j}, \vec{c} = a\hat{k}, \] where \( \hat{i}, \hat{j} \) and \( \hat{k} \) are unit vectors in \( X, Y \) and \( Z \) axes, \( a \) be the magnitude of each vectors. The volume of the cubic cell

\[ V = \vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b}) = a^3 \]
The primitive translational vectors of the reciprocal simple cubic lattice are given by:

\[
\vec{A} = 2\pi \frac{\hat{b} \times \hat{c}}{\hat{a} \cdot (\hat{b} \times \hat{c})} = 2\pi \frac{aj\times ak}{a^3} = \frac{2\pi}{a} \hat{i}
\]

\[
\vec{B} = 2\pi \frac{\hat{c} \times \hat{a}}{\hat{b} \cdot (\hat{c} \times \hat{a})} = 2\pi \frac{ak\times ai}{a^3} = \frac{2\pi}{a} \hat{j}
\]

\[
\vec{C} = 2\pi \frac{\hat{a} \times \hat{b}}{\hat{c} \cdot (\hat{a} \times \hat{b})} = 2\pi \frac{ai\times aj}{a^3} = \frac{2\pi}{a} \hat{k}
\]

Thus, we find that the reciprocal lattice to the sc direct space lattice is itself a simple cubic lattice with lattice constant \(\frac{2\pi}{a}\).

(i) The primitive translational vectors say \(\hat{a}', \hat{b}', \text{ and } \hat{c}'\) of the body centred cubic (bcc) lattice are given by

\[
\hat{a}' = \frac{a}{2} (i + j - k)
\]

\[
\hat{b}' = \frac{a}{2} (-i + j + k)
\]

\[
\hat{c}' = \frac{a}{2} (i - j + k)
\]

The volume of the primitive cell is

\[
V = \hat{a}' \cdot (\hat{b}' \times \hat{c}') = \hat{b}' \cdot (\hat{c}' \times \hat{a}') = \hat{c}' \cdot (\hat{a}' \times \hat{b}')
\]

\[
\hat{b}' \times \hat{c}' = \frac{a^2}{2} (i + j)
\]

\[
\therefore V = \frac{a}{2} (i + j - k) \cdot \frac{a^2}{2} (i + j) = \frac{a^3}{2}
\]

Now, the primitive translational vectors of the reciprocal lattice are given by

\[
\vec{A} = 2\pi \frac{\hat{b}' \times \hat{c}'}{\hat{a}' \cdot (\hat{b}' \times \hat{c}')} = 2\pi \frac{\frac{a^2}{2}(i+j)}{a^3/2} = \frac{2\pi}{a} (i + j)
\]

Similarly,

\[
\vec{B} = 2\pi \frac{\hat{c}' \times \hat{a}'}{\hat{b}' \cdot (\hat{c}' \times \hat{a}')} = 2\pi \frac{\frac{a^2}{2}(j+i)}{a^3/2} = \frac{2\pi}{a} (j + i)
\]

\[
\vec{B} = 2\pi \frac{\hat{a}' \times \hat{b}'}{\hat{c}' \cdot (\hat{a}' \times \hat{b}')} = 2\pi \frac{\frac{a^2}{2}(k+i)}{a^3/2} = \frac{2\pi}{a} (j + k)
\]

Thus, the reciprocal bcc lattice vectors are the primitive vectors of \(fcc\) lattice with lattice constant \(2\pi/a\).

(ii) The primitive translational vectors of the face centred cubic lattice are given by:

\[
\hat{a}' = \frac{a}{2} (i + j)
\]

\[
\hat{b}' = \frac{a}{2} (j + k)
\]
\[ \vec{c}' = \frac{a}{2} (\hat{k} + \hat{i}) \]

The volume of the primitive cell is given by
\[ V = \vec{a}' \cdot (\vec{b}' \times \vec{c}') = \vec{b}' \cdot (\vec{c}' \times \vec{a}') = \vec{c}' \cdot (\vec{a}' \times \vec{b}') \]

Here,
\[ \vec{b}' \times \vec{c}' = \frac{a^2}{4} (\hat{i} + \hat{j} - \hat{k}) \]
∴ \[ V = \frac{a^2}{2} (\hat{i} + \hat{j}) \cdot \frac{a^2}{4} (\hat{i} + \hat{j} - \hat{k}) = \frac{a^3}{4} \]

Thus, the primitive translational vectors of the reciprocal fcc lattice are given by:
\[ \vec{A} = 2\pi \left( \frac{\vec{b}' \times \vec{c}'}{\vec{a}' \cdot (\vec{b}' \times \vec{c}')} \right) = 2\pi \frac{a^2}{a^3} (\hat{i} + \hat{j} - \hat{k}) \]
\[ \vec{B} = \frac{2\pi}{a} (-\hat{i} + \hat{j} + \hat{k}) \]
\[ \vec{C} = \frac{2\pi}{a} (\hat{i} - \hat{j} + \hat{k}) \]

Thus, we have shown that the reciprocal fcc lattice are just the primitive vectors of bcc lattice with lattice constant \( \frac{2\pi}{a} \).

10. Calculate the Einstein temperature given Einstein frequency as \( 9 \times 10^{11} \) Hz. Given \( k = 1.4 \times 10^{23} J/K^{-1} \).
Hint: Use Einstein temperature formula \( \theta_E = \frac{h \nu}{k} \).
Ans: 43K.

11. Calculate the characteristic temperature \( \theta_E \) for copper if Einstein frequency \( \nu_F = 4.8 \times 10^{12} \) Hz.
Ans: 226K

12. Calculate the room temperature specific heat capacity if Debye temperature of the solid is 3000K?
Hint: Use \( C_v = \frac{12}{5} \pi^4 \frac{R}{T_D^3} \), where room temperature can be taken as \( T = 300K \) and \( R = 8.314 \frac{J}{mol\cdot K} \)
Ans. 1.93 \( \frac{J}{mol\cdot K} \)

13. Gold has the same structure as copper. The velocity of sound in gold is 2100 m/s and that in copper is 3800 m/s. If the Debye temperature of Gold is 170 K, determine the Debye temperature of copper. Density of gold= 1.93 \( \times 10^4 \) kg/m\(^3\) and density of copper= 8.96 \( \times 10^3 \) kg/m\(^3\). Atomic weight of gold= 197.0 atomic weight of copper= 63.54 a. m. u.
Solution: The Debye temperature \( \theta_D = \frac{h \nu_D}{k} \), where \( \nu_D \) is called Debye frequency given by
\[ \nu_D^3 = \frac{9N}{4\pi^2} \left[ \frac{1}{V_i^3} + \frac{2}{V_t^3} \right]^{-1} \]
where \( N \) = number of atoms in a volume \( V \)
\[ v_l = \text{Longitudinal velocity of sound waves in the crystal} \]

\[ v_t = \text{Transverse velocity of sound waves in the crystal} \]

We can take nearly both components to be equal i.e. \( v_l = v_t = v \). Hence:

\[ \Rightarrow V_D^3 = \frac{9N \nu^3}{4\pi V} \]

\[ v_D = \nu \left( \frac{3N}{4\pi V} \right)^{1/3} \]

But \( \theta_D = \frac{\hbar V_D}{k} = \frac{\hbar}{k} \nu \left( \frac{3N}{4\pi V} \right)^{1/3} \)

We know, atomic mass \( M \) (1 mole mass) contains number of atoms = \( N_A \) (Avogadro’s number)

Or,

\[ m \text{ mass contains number of atoms} = \frac{N_A}{M} \times m \]

\[ \therefore \frac{N}{V} = \frac{N_A}{M} \times \rho, \rho \text{ be the density of the solid.} \]

Thus

\[ \frac{\theta_D (Cu)}{\theta_D (Au)} = \frac{h \nu_{Cu} \left( \frac{3N_A}{4\pi m_{Cu}} \times \rho_{Cu} \right)^{1/3}}{h \nu_{Au} \left( \frac{3N_A}{4\pi m_{Au}} \times \rho_{Au} \right)^{1/3}} = \frac{2100 \left( \frac{3800}{197} \times 8.96 \times 10^3 \right)^{1/3}}{63.54 \times 1.93 \times 10^4} = 1.80 \times 1.13 \]

\[ \Rightarrow \theta_D (Cu) = \theta_D (Au) \times 2.29 = 170 \times 2.04 = 348 K \]

14. The energy of two particles in the field of each other is given by

\[ U = -\frac{\alpha}{r^2} + \frac{\beta}{r^{10}} \], where \( \alpha \) and \( \beta \) are constants and \( r \) be the separation between the particles.

Determine the separation between the particles for a stable compound and also determine the cohesive energy of the crystal.

Solution:

We have the energy of the two particles:

\[ U = -\frac{\alpha}{r^2} + \frac{\beta}{r^{10}} \]

For a stable compound, let the separation between the particles be \( r_0 \) where \( \frac{dU}{dr} \bigg|_{r=r_0} = 0 \)

Then,

\[ \frac{dU}{dr} = \frac{2\alpha}{r^3} - \frac{10\beta}{r^{11}} = 0 \]

Or, \( r^8 = \frac{5\beta}{\alpha} \)

Or, \( r = \left( \frac{5\beta}{\alpha} \right)^{1/8} = r_0 \)

Putting the value of \( r_0 \) in above equation

\[ U_{min} = -\alpha \left( \frac{\alpha}{5\beta} \right)^{1/4} + \beta \left( \frac{\alpha}{5\beta} \right)^{5/4} \]

\[ = \frac{\alpha^{5/4}}{(5\beta)^{1/4}} \left( \frac{\beta}{5\beta} - 1 \right) = -\frac{4}{5} \left( \frac{\alpha^{5/4}}{(5\beta)^{1/4}} \right) = -\frac{4\alpha}{5r_0^2} \]

This is the Cohesive energy of the crystal.
15. Show that average kinetic energy of a free electron at 0K is $\frac{3}{5} E_f$, where $E_f$ is the Fermi energy.

Solution:
Average kinetic energy of a free electron is given by

$$\langle E_0 \rangle = \frac{1}{N} \int_0^{E_f} E \, f(E) \, dE$$

where $D(E)$ be the density of states and $f(E)$ be the fermi function.

At absolute zero all the energy states are completely occupied up to fermi energy $E_f$. Hence, the fermi function

$$f(E) = \frac{1}{e^{(E-E_f)/K_B T} + 1} = 1$$, since $E < E_f$ at absolute zero.

Density of states

$$D(E) = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$$

Thus, equation (1) becomes:

$$\langle E_0 \rangle = \frac{1}{N} \int_0^{E_f} E \, \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} \, dE$$

$$= \frac{1}{N} \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^{E_f} E^{3/2} \, dE$$

$$= \frac{1}{N} \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \left(\frac{2}{5} E_f^{5/2}\right)_0^{E_f}$$

$$= \frac{1}{N} \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \frac{2}{5} E_f^{5/2}$$

But, $N = \frac{V}{3\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} E_f^{3/2}$ at fermi energy.

So, $\langle E_0 \rangle = \frac{3}{5} E_f$

16. There are $2.5 \times 10^{28}$ free electrons per cubic meter of Sodium. Calculate the Fermi energy and Fermi velocity.

Hint: Use the relations:

$$E_F = \frac{h^2}{2m} \left(\frac{3\pi^2 N}{V}\right)^{2/3}$$ for fermi energy and $v_F = \frac{h}{m} \left(\frac{3\pi^2 N}{V}\right)^{1/3}$ for fermi velocity.

where $N = 2.5 \times 10^{28}$ m$^{-3}$ and $m = 9.1 \times 10^{-31}$ kg.

Ans: $4.97 \times 10^{-19}$ Joule and $1.0 \times 10^6$ m/s.

17. Consider silver in the metallic state with one free electron per atom. Calculate the fermi energy.

Given density of silver is 10.5 gm/cm$^3$ and atomic weight 108.

Hint: Use $E_F = \frac{h^2}{2m} \left(\frac{3\pi^2 N}{V}\right)^{2/3}$, where $N = \rho N_A / M = \frac{10.5 \times 6.023 \times 10^{23}}{108} = 5.85 \times 10^{28}$ m$^3$.

Ans. $8.77 \times 10^{-19}$ Joule.

18. Aluminum metal crystallises in $fcc$ structure. If each atom contributes single electron as free electron and the lattice constant $a$ is 4 Å. Calculate treating conduction electrons as free electron fermi gas: (i) Fermi energy (ii) fermi vector (iii) Total kinetic energy of free electron gas per unit volume at 0 K.

Hint: Use $E_F = \frac{h^2}{2m} \left(\frac{3\pi^2 N}{V}\right)^{2/3}$, where $N = $ number of electrons per unit cell = 4 for $fcc$ crystal and $V = a^3 = (4 \times 10^{-10})^3 = 64 \times 10^{-30}$ m$^3$.

$\therefore E_f = 9.19 \times 10^{-19}$ Joule (Ans.)

We have the relation for fermi vector, $K_f = \left(\frac{3\pi^2 N}{V}\right)^{1/3}$ $\therefore E_f = \frac{h^2}{2m} K_f^2$
Ans. 1.27 × 10^{10} m^{-1}

Total kinetic energy per unit volume at 0K = Average K.E. × number density
\[ = \frac{3}{2}E_F \times \frac{N}{V} = 21.52 \times 10^{28} eV \]

19. Find the fermi energy in copper on the assumption that each copper atom contributes one free electron to the electron gas. The density of copper is 8.94 × 10^3 kg/m^3 and its atomic mass = 63.5 a.m.u.

Hint: Use \[ E_F = \frac{h^2}{2m} \left( \frac{3\pi^2N}{V} \right)^{2/3}, \text{where} \frac{N}{V} = \frac{\rho N_A}{M} = 8.48 \times 10^{28} / m^3, m = 9.1 \times 10^{-31} \text{kg}, \]

Ans. 11.3 × 10^{-19} Joule.

20. If the fermi energy of a metal is 10 eV, What is the corresponding classical temperature?

Hint: Use \[ E_F = K_BT_f \], where \( K_B = 1.38 \times 10^{-23} J/K \)

Ans. \( T_f = 1.16 \times 10^5 K \)

21. An insulator has an optical absorption which occurs for all wavelengths shorter than 1800 Å. Find the width of the forbidden energy band for this insulator.

Hint: The minimum energy required to move the electron from valence band to conduction band is the energy equal to the light of wavelength 1800 Å because the insulator can absorb only wavelength shorter than 1800 Å. Thus, energy gap of the forbidden band
\[ E = h\nu = h\frac{c}{\lambda} = 6.62 \times 10^{-34} \times 3 \times \frac{10^8}{1800 \times 10^{-10}} = 1.1 \times 10^{-18} = 6.89 \text{ eV} \]

22. Assuming that each atom of copper contributes one free electron, calculate the drift velocity of free electrons in the copper conductor of cross-sectional area 10^{-4} m^2 carrying a current of 200 A. Given atomic weight of copper = 63.5, density of copper = 8.94 × 10^3 kg/m^3, \( e = 1.602 \times 10^{-19} C \).

Hint: Drift current density due to electrons
\[ J_e = n_e e v_d \], where \( n_e \) = number of electrons per unit volume, \( v_d \) = drift velocity of electrons.

Since each atom contributes one free electron,
\[ n_e = \text{number of atoms per unit volume} = \frac{\rho N_A}{M} = \frac{8.94 \times 10^3 \times 6.023 \times 10^{26}}{63.5} = 8.48 \times 10^{28} / m^3 \]

Current density \[ J_e = \frac{I}{a} = \frac{200}{10^{-4}} = 2 \times 10^6 A \text{ m}^{-2} \]

\( \therefore \) Drift velocity \( v_d = \frac{J_e}{n_e e} = \frac{2 \times 10^6}{8.48 \times 10^{28} \times 1.602 \times 10^{-19}} = 1.47 \times 10^{-4} \text{ m/s} \)

23. Calculate the conductivity of germanium. Given mobilities of electrons and holes in a sample of germanium at room temperature are 0.54 m^2 V^{-1} s^{-1} and 0.18 m^2 V^{-1} s^{-1} respectively.

Assuming that electron and hole densities are each equal to 3.6 × 10^{19} per m^3. If the potential difference of 2 volts is applied across the germanium plate of thickness 0.2 mm and area 1 cm^2, calculate the current placed in the plate.

Hint: Conductivity of an intrinsic semi-conductor is given by
\[ \sigma = e n_i (\mu_e + \mu_h) \]
where \( e = 1.6 \times 10^{-19} C, n_i = 3.6 \times 10^{19} \text{ per m}^3, \mu_e = 0.54 \text{ m}^2 \text{V}^{-1} \text{s}^{-1} \) and \( \mu_h = 0.18 \text{ m}^2 \text{V}^{-1} \text{s}^{-1} \)

\( \therefore \) \( \sigma = 1.6 \times 10^{-19} \times 3.6 \times 10^{19} (0.54 + 0.18) = 4.15 \)

\( \rho = \frac{1}{\sigma} = \frac{1}{4.15} = 0.24 \text{ ohm meter} \)

We know: \( R = \frac{\rho l}{A} \), where \( l = 0.2 \text{ mm} = 0.2 \times 10^{-3} \text{m} \), \( A = 1 \text{ cm}^2 = 10^{-4} \text{m}^2 \)

\( \therefore R = \frac{0.24 \times 0.2 \times 10^{-3}}{10^{-4}} = 0.48 \text{ ohm} \)
Applied voltage $E = 2V \therefore I = \frac{E}{R} = \frac{2}{0.48} = 4.17$ Ampere.

24. The transition temperature of mercury with an average atomic mass of 200.59 a. m. u. is 4.153 K. Determine the transition temperature of one its isotopes $Hg_{204}^{304}$.

Hint: The transition temperature of super conductor and its isotopic mass $M$ are connected by the relation 

$$T_c \propto \frac{1}{M^{1/2}}$$

$$\therefore \frac{T_c_2}{T_c_1} = \left(\frac{M_1}{M_2}\right)^{1/2} = \sqrt{\frac{200.59}{204}}$$

$$\Rightarrow T_c_2 = 4.153 \times \sqrt{\frac{200.59}{204}} = 4.118 K$$

25. The critical field of 6K and 8K for a superconducting alloy are 7.616 mAm$^{-1}$ and 4.284 mAm$^{-1}$ respectively. Determine the critical temperature and critical field at 0K.

Hint: Use the relation

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$H_{c1} = 7.616$ mAm$^{-1}$ and $H_{c2} = 4.284$ mAm$^{-1}$

Then

$$7.616 = H_0 \left[1 - \left(\frac{6}{T_c}\right)^2\right]$$

and

$$4.284 = H_0 \left[1 - \left(\frac{8}{T_c}\right)^2\right]$$

Solving these equations: we get $T_c = 10K$

Now, $7.616 = H_0 \left[1 - \left(\frac{6}{10}\right)^2\right]$

Or, $7.616 = H_0 \left[1 - \left(\frac{6}{10}\right)^2\right]$

$$\Rightarrow H_0 = 11.9 \text{ mAm}^{-1}$$

26. A super conductor specimen has a critical temperature of 3.7 K in zero magnetic field and a critical field of 0.03 Tesla at 0K. Find the critical field at 2K.

Hint: Use $H_c = H_0 \left[1 - \left(\frac{T}{T_c}\right)^2\right]$, where $T_c = 3.7 K$ and $H_0 = 0.03$ Tesla, $T = 2K$. 

Ans:$H_c = 0.021$ Tesla.

27. The average energy required to create a vacancy in a certain metal is 1 eV. Calculate the ratio of vacancies in metal at 600K and 300K.

Solution: The number of vacancies in metal is given by:

$$n = N e^{-\frac{E_v}{k_BT}}$$

where average energy $E_v = 1 eV = 1.6 \times 10^{-19} J, k_B = 1.38 \times 10^{-23} J/K$. Then,

$$n_{600} = \frac{e^{-\frac{1.6\times10^{-19}}{600\times1.38\times10^{-23}}}}{e^{-\frac{1.6\times10^{-19}}{300\times1.38\times10^{-23}}}} = 2.46 \times 10^8$$

28. The resistivity of copper at 273K is $1.55 \times 10^{-6}$ ohm cm. Estimate the electronic thermal conductivity of copper at the same temperature.

Solution: We have given $T = 273K, k_B = 1.38 \times 10^{-23} J/K, e = 1.6 \times 10^{-19} C$, resistivity $\rho = 1.55 \times 10^{-6}$ ohm cm. We know the relation for electronic thermal conductivity.
\[
K_{el} = \left( \frac{\pi^2 n k_B^2 T}{3 m} \right) T, \text{ where the relaxation time is given by } \tau = \frac{m}{ne^2 \rho}. \text{ Combining both equations:}
\]
\[
K_{el} = \left( \frac{\pi^2 n k_B^2 T}{3 m} \right) m = \frac{\pi^2 k_B^2 T}{3 e^2 \rho} = \frac{\pi^2 (1.38 \times 10^{-23})^2 \times 273}{3 \times (1.6 \times 10^{-19})^2 \times 1.55 \times 10^{-8}} = 430.6 \text{ W} K^{-1} m^{-1}.
\]

29. The resistivity of intrinsic germanium at 300K is 47 Ohm-cm. Calculate the intrinsic carrier density of germanium at the same temperature, given that the electron and hole mobilities are 0.39 m²/v⁻¹s⁻¹ and 0.19 m²/v⁻¹s⁻¹ respectively. [Electronic charge \( e = 1.6 \times 10^{-19} \text{ C} \).]

Hint: Use \( \sigma = \frac{1}{\rho} \), where \( \rho = \frac{1}{\sigma} = 47 \text{ ohm - cm} = 0.47 \text{ ohm - m} \).

\[ n_i = \text{intrinsic carrier density} = \frac{1}{\rho (\mu_e + \mu_h)} = \frac{1}{0.47 \times 1.6 \times 10^{-19} \times (0.39 + 0.19)} = 2.292 \times 10^{19} \text{ m}^{-3}. \]

30. Show that crystal lattice can not have five fold symmetry.

Solution: Let us consider two lattice points A and B with positions vectors \( \vec{r}_1 \) and \( \vec{r}_2 \) respectively. Let \( \vec{AB} = \vec{a} \) be a primitive translation vector, then

\[ |\vec{r}_1 - \vec{r}_2| = |\vec{a}| \]

Let the vector \( \vec{AB} = \vec{a} \) be rotated about a line perpendicular to the plane of the paper and passing through the point A with an angle \( \phi = \frac{2\pi}{n} \) such that the new lattice vectors \( \vec{OA}' = \vec{r}_1' \). Similarly, let the vector \( \vec{a} \) be rotated about an axis passing through the lattice point B and perpendicular to the plane of the paper through the same angle \( \phi = \frac{2\pi}{n} \) such that the new lattice point \( B' \). Let \( \vec{OB}' = \vec{r}_2' \) as shown in figure.

Here \( A'B' = m|\vec{a}| \), where m is an integer. Draw perpendiculars AC and BD from points A and B on the line \( A'B' \), then we can write

\[ A'B' = m|\vec{a}| = A'C + CD + DB' = |\vec{a}| \sin \left( \phi - \frac{\pi}{2} \right) + |\vec{a}| + |\vec{a}| \sin \left( \phi - \frac{\pi}{2} \right) \]

\[ \Rightarrow m = 1 + 2 \sin \left( \phi - \frac{\pi}{2} \right) = 1 + 2 \cos \phi \]

Or, \( \cos \phi = \frac{1 - m}{2} \)

When \( m = -2 \), then \( \cos \phi = \frac{3}{2} \) which is not possible \( \because -1 \leq \cos \phi \leq 1 \)

When \( m = -1 \), then \( \cos \phi = 1 \Rightarrow \phi = \frac{2\pi}{4} \)

When \( m = 0 \), then \( \cos \phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3} = \frac{2\pi}{6} \)

When \( m = 1 \), then \( \cos \phi = 0 \Rightarrow \phi = \frac{\pi}{2} = \frac{2\pi}{4} \)

When \( m = 2 \), then \( \cos \phi = -\frac{1}{2} \Rightarrow \phi = \frac{2\pi}{3} \)
When $m = 3$, then $\cos \phi = -1 \Rightarrow \phi = \frac{2\pi}{2}$

When $m = 4$, then $\cos \phi = -\frac{3}{2}$, which is not possible again.

So, the possible values of $\phi$ are only $\frac{2\pi}{1}, \frac{2\pi}{2}, \frac{2\pi}{3}, \frac{2\pi}{4}$ and $\frac{2\pi}{6}$ but not $\frac{2\pi}{5}$ or $n = 1, 2, 3, 4$ and 6 only.

Hence, $n = 5$ is not possible. Therefore, fivefold rotation is not possible in an ideal crystal.

31. Show that the volume of direct lattice is inversely proportional to the volume of reciprocal lattice.

Solution: If $\vec{a}, \vec{b}$ and $\vec{c}$ be the primitive translation vectors of direct lattice and $\vec{A}, \vec{B}$ and $\vec{C}$ are that for reciprocal lattice, then volume of solid in terms of reciprocal lattice is

$$V = \vec{A} \times \vec{B} = \frac{8\pi^3 (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) \vec{c} \times (\vec{a} \times \vec{b})}{|a \cdot b \cdot c|^3}$$

$$= \frac{8\pi^3 (\vec{b} \times \vec{c}) \cdot [(\vec{c} (\vec{a} \times \vec{b}) \vec{a} - \vec{a} (\vec{a} \times \vec{b})) \vec{c}]}{|a \cdot b \cdot c|^3}$$

$$= \frac{8\pi^3 |a \cdot b \cdot c|^2}{|a \cdot b \cdot c|^3}$$

$$\Rightarrow V \propto \frac{1}{V'}$$

32. Estimate the diamagnetic susceptibility of argon. Argon has atomic number 18 and at temperature $4K$, its concentration is $2.66 \times 10^{28}$ m$^3$. Take root mean square distance of an electron from nearest nucleus to be 0.66Å. Also calculate the magnetization of solid argon at $2T$.

Hint: Use the relation for volume susceptibility of the material:

$$\chi = -\frac{\mu_0 N e^2}{6m} \langle R^2 \rangle$$, where $N = 2.66 \times 10^{28}$ m$^3$, $Z = 18$, $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg, for an electron, $\mu_0 = 4\pi \times 10^{-7}$ henry/meter, $\sqrt{\langle R^2 \rangle} = 0.66Å = 0.66 \times 10^{-10}$ m.

The relation for corresponding magnetization:

$$M = -\frac{NZe^2}{6m} \langle R^2 \rangle$$, where $B = 2T$.

Ans: $-12.28 \times 10^{-6}$ (unitless) and $-19.54$ Ampere/meter.

33. A paramagnetic substance has $10^{28}$ atm/m$^3$. The magnetic moment of each atom is $1.8 \times 10^{-23}$ ampere meter$^2$. Calculate the paramagnetic susceptibility at $300K$. What would be the dipole moment of a bar of this material 0.1 meter long and 1sq.cm cross-section placed in a field of $8 \times 10^4$ amp/meter.

Hint: Use the relation for paramagnetic susceptibility

$$\chi = \frac{n m^2 \mu_0}{3kT}$$, where $n = 10^{28}$ atm/m$^3$, magnetic moment $m = 1.8 \times 10^{-23}$ Am$^2$,

$\mu_0$ = permeability of vacuum $= 4\pi \times 10^{-7}$ Henry/meter, Boltzmann constant $= 1.38 \times 10^{-23}$ J/K, $T = 300K$

Then $\chi = \frac{n m^2 \mu_0}{3kT} = \frac{10^{28} \times (1.8 \times 10^{-23})^2 \times 4\pi \times 10^{-7}}{3 \times 1.38 \times 10^{-23} \times 300} = 3.3 \times 10^{-4}$

Since magnetization $M = \chi H$, where $H$ is called applied magnetizing field. Here $H = 8 \times 10^4$ amp/meter.

Thus, $M = \chi HL$ where $L =$ length of the bar.
Now dipole moment $\sum m = V \times M$, where $V =$ volume = $0.1 \times 10^{-4}$, where we have used cross-sectional area= $10^{-4}m^2$
\[\therefore \sum m = 0.1 \times 10^{-4} \times 3.3 \times 10^{-4} \times 8 \times 10^4 \times 0.1 = 2.65 \times 10^{-5} \text{A} - m^2.\]

34. A magnetic material has a magnetization of 3300 amp/meter and flux density of 0.0044 wb/m$^2$. Calculate the magnetizing force and relative permeability of the material.

Hint: Use the relation:
\[B = \mu_0 (M + H)\]
where $M = 3300 \text{amp/meter}$ and $B = 0.0044 \text{wb/m}^2$ and $\mu_0 = 4\pi \times 10^{-7} \text{H/m}$.

Thus, $H = \frac{B}{\mu_0} - M = \frac{0.0044}{4\pi \times 10^{-7}} - 3300 = 201.4 \text{amp/meter}$.

We can also write:
\[M = H(\mu_r - 1), \text{ where } \mu_r \text{ is called relative permeability.}\]
\[\therefore \mu_r = \frac{M}{H} + 1 = \frac{3300}{201.4} + 1 = 17.38\]

35. Calculate the magnitude of hall voltage induced across a silver wire of square cross section 1mm on a side, when it carries a current of 1.5 A and a transverse magnetic field of strength 0.1T is applied. Free electron density is $5.85 \times 10^{28} \text{m}^{-3}$.

Hint: Use the relation of Hall electric field:
\[E_y = \frac{I}{ne} B_z. \text{ Since electric field} = \frac{\text{voltage}}{\text{distance}} \text{ and current density} = \frac{\text{current(l)}}{\text{cross-sectional area}(a)^2}, \]
\[\therefore \frac{V_H}{l} = \frac{I B_z}{a ne}, \]
where $I = 1.5A, B_z = 0.1 \text{T}, n = 5.85 \times 10^{28} \text{m}^{-3}, e = 1.6 \times 10^{-19} \text{C}, l = 1 \text{mm} = 10^{-3} \text{m}$
\[\therefore \frac{V_H}{l ne} = \frac{I B_z}{l^2 ne} \text{, where } a = l^2\]
\[\therefore \frac{V_H}{l ne} = \frac{1.5 \times 0.1}{10^{-3} \times 5.85 \times 10^{28} \times 1.6 \times 10^{-19}} = 1.60 \times 10^{-8} \text{V} \]